Executive summary

Many institutional investors with long time horizons have been increasing allocations to alternatives, particularly illiquid investments. We would agree that illiquid investments are an important part of the overall opportunity set of an appropriately diversified portfolio. However, the financial crisis has highlighted how critical it is for investors to account for the increased liquidity risk in their asset allocation framework and the way it could affect the overall portfolio – from the impact of capital calls to the extreme possibility of insolvency.

In this paper, we develop key insights into how liquidity risk is manifest in illiquid investments, and guidelines to help manage that risk within the overall portfolio construction process:

- A stochastic modeling approach is better suited to capturing the potential impact of liquidity risk: to address liquidity risk more effectively, we suggest that investors move beyond a mean-variance framework and incorporate projected liabilities and constraints into the decision-making process. We demonstrate an approach to incorporating expected cashflows into the asset allocation process. This allows the investor to account for the timing of cashflows associated with an illiquid investment as well as the potential for a forced selling scenario (in which a secondary market transaction would likely be at a discount to net asset value (NAV)).

- The traditional mean-variance framework overstates the optimal allocation to illiquid assets: incorporating the dynamics of cashflows associated with illiquid assets results in lower efficient allocations to illiquid assets, compared with traditional mean-variance allocations. The effect is particularly pronounced at high expected risk levels.

- Our analysis is based on the inclusion of private equity into a “traditional” portfolio comprising bonds and public equities. However, we believe similar considerations apply to other illiquid investments, albeit with reduced constraints for assets that generate regular cashflows.
Optimal allocation to illiquid assets varies with the investor’s ability to meet cash obligations: we provide guidelines for an appropriate allocation to illiquid investments that is based on the investor’s ability to meet existing cash obligations.

Our guidelines for investors are driven by three primary factors: the expected return premium of illiquid assets over liquid assets, the discount to NAV for secondary market transactions, and the timing of distributions from illiquid investments. For investors with low-to-moderate portfolio risk levels, the expected timing of distributions from the illiquid investment has the largest impact on the optimal allocation to illiquid assets. For investors with higher levels of portfolio risk, the expected return premium and the assumed discount to NAV in the event of a secondary sale have the greatest impact on the optimal allocation.

We thank Wilson Lee, Catherine LeGraw, Marcia Roitberg, Brianna Barrett, David Greenberg, Ronald Kahn, Jason Malinowski, Tom McFarren, Julia Wittlin, John Dewey, Fons Lute, Ian Kitchenham, Michelle Dunne, Edward Fishwick and Chris Wray for their contributions to this paper.
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Introduction

Institutional investors have been increasing allocations to illiquid investments such as private equity, private real estate, and real assets (including timberland, farmland, infrastructure and energy). Because these investors typically have relatively long-dated liabilities, they often have the ability to forgo current liquidity for the potential to earn higher returns by investing in less liquid assets.

Illiquid investments are an important component of the investment opportunity set. They may offer a differentiated risk profile and the potential for higher returns than publicly traded assets. For example, US private equity buyout investments have a historic return premium of 3–5% over public equities, after controlling for differences in market risk (Malinowski 2010).

Prudent investment in illiquid assets requires that investors consider liquidity risk during the asset allocation process. The financial crisis underscored this point, when a number of investors struggled to access sufficient cash to meet spending obligations. The price decline of liquid assets – combined with lower-than-expected distributions from private investments – resulted in large proportionate increases in the illiquid asset portfolios. The aftermath left many investors to reassess the potential impact of liquidity risk on their portfolios.

The challenge in assessing liquidity risk is in the timing of drawdowns and distributions, along with the fact that liability requirements are not easily modeled in a traditional mean-variance framework. To help investors better assess and manage liquidity risk, we address the following questions:
How can investors incorporate liquidity risk in the portfolio construction process?

What are the critical factors in determining the optimal allocation (or capacity) for illiquid investments?

What is an appropriate allocation to illiquid assets?

We propose a framework for assessing the liquidity risk of an illiquid portfolio and incorporating this risk into the portfolio construction process. We begin by identifying the forms of liquidity risk and defining a simple three-asset portfolio to demonstrate the framework. Next, we look at the portfolio given two different approaches to asset allocation – traditional mean variance and stochastic modeling. We then determine how the optimal illiquid allocation varies along with spending requirements. Lastly, we look at the three key assumptions of our framework (the expected return premium, the discount to NAV and the timing of cashflows) and consider how these may impact optimal portfolio allocations.
Practitioners and academics have different definitions for liquidity risk. Hibbert et al. (2009) distinguish trading liquidity (the ease with which an asset can be traded) from funding liquidity (a firm’s access to funding).

Trading liquidity can be estimated from bid-ask spreads and market depth, as well as volumes and frequency of transactions. Trading liquidity risk, therefore, relates to the risk of increased difficulty of transacting in an asset. This can be apparent in an environment of widespread selling caused by technical factors. Examples include the flash crash of May 2010 or the dislocated convertible bond market in 2008.

Funding liquidity relates to the ability to finance transactions, including margin, collateral or capital commitments. Risks to the availability of funding can be seen in repo rates as well as margin and collateral requirements. In illiquid assets, particularly private equity, funding liquidity risk relates to the ability to meet outstanding capital commitments.

These two risks are related, as one way to mitigate funding liquidity risk is by having the ability to source capital from more liquid parts of the portfolio. A portfolio of public market assets has relatively low trading liquidity risk (securities can be sold on the secondary market without substantial transaction costs, in most market conditions) and relatively low funding liquidity risk (financing terms tend to be more favorable). Increased liquidity risk also carries a hidden cost in the form of decreased flexibility to rebalance a portfolio (due to a change in the investor’s views or a need to rebalance back to the policy weights), which may translate into an opportunity cost.

For the purposes of this paper, we define liquidity risk as the increased likelihood of a liquidity event. We define a liquidity event as the forced sale of illiquid assets at a discount due to insufficient capital available to make required payments from liquid assets. Liquidity risk increases in two ways: when it becomes challenging to find counterparties who are willing to buy the illiquid allocation, requiring transactions to be completed at a discount to NAV (trading risk); and when drawdowns of committed capital impair the ability to make liability payments or other spending requirements (funding risk).

Traditional mean-variance approaches to asset allocation do not factor in the particular dynamics associated with illiquid investing.

For simplicity, throughout this paper we consider a three-asset portfolio composed of fixed income, public equity and private equity (our representative illiquid asset), using the capital market assumptions defined by Malinowski et al. (2012). This allows us to focus on the key factors that affect an allocation decision with illiquid characteristics, without being wedded to a particular set of asset classes. As a result, the portfolio...
allocations derived are illustrative; in practice, investors will have many additional factors for consideration as well as a broader opportunity set. Our capital market assumptions are defined in Figure 1.

**FIGURE 1: CAPITAL MARKET ASSUMPTIONS**

<table>
<thead>
<tr>
<th></th>
<th>Expected total return</th>
<th>Expected risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed income</td>
<td>2.5%</td>
<td>4.5%</td>
</tr>
<tr>
<td>Public equity</td>
<td>7.5%</td>
<td>17.0%</td>
</tr>
<tr>
<td>Private equity</td>
<td>10.5%</td>
<td>30.0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Fixed income</th>
<th>Public equity</th>
<th>Private equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed income</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public equity</td>
<td>0.1</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Private equity</td>
<td>0.2</td>
<td>0.8</td>
<td>1.0</td>
</tr>
</tbody>
</table>

| Expected inflation | 2.0% |

Source: BlackRock, as of August 2013. Capital market assumptions assume a full market cycle.

**CASHFLOW-INDIFFERENT VS. CASHFLOW-SENSITIVE MODELING**

We begin the analysis by conducting a traditional mean-variance optimization to provide a starting point for evaluation. In the mean-variance approach to asset allocation, expected return and risk characteristics are defined for each asset class, and an investor’s asset allocation is determined by their willingness to take on risk in exchange for return at the portfolio level.

Using the traditional mean-variance approach, asset allocation across the efficient frontier is shown for a long-only, fully invested and unlevered portfolio (Figure 2). The allocations for private equity represent drawn or invested capital, as opposed to committed capital. Notice that as portfolio volatility increases, fixed income allocations decrease and private equity allocations increase. Public equity allocations also increase to a point, after which they are substituted for private equity, which is assumed to have higher expected return and volatility characteristics. This shift is consistent with intuition; in order to achieve a higher expected return, the portfolio must take on higher volatility by increasing the allocation to private equity. Representative optimal allocations at 10% and 15% portfolio risk are highlighted for illustrative purposes, because institutional portfolios often have an expected total risk of 10–15%.

1 Risk and return data for private equity is adjusted slightly from Malinowski et al. (2012), to reflect changing capital market assumptions.

2 After fixed income allocations drop off from the efficient frontier, private equity becomes the dominant asset in the higher volatility portfolios, due to its strong correlation with public equity and higher relative return.
Alternatives and Liquidity: Incorporating Liquidity Constraints into Portfolio Construction

Mean-variance optimization assumes all relevant asset allocation information is captured by return, volatility, and correlation forecasts. Cashflow dynamics and the evolution of the portfolio over time are difficult to accurately capture without additional information in a mean-variance optimization. A penalty function representing disutility from allocating to illiquid assets may be used. Another recent approach discussed by Meucci (2012) is to enhance the risk estimate in a "liquidity-plus-market-risk distribution".

What is liquidity risk?

SECTION 1

FIGURE 2: INCREASES IN RISK CORRESPOND TO INCREASES IN ILLIQUID ASSETS AND EXPECTED RETURNS

Mean-variance approach: Optimal allocations

However, the traditional mean-variance approach does not effectively model the dynamics associated with investing in illiquid assets. Specifically, it excludes the timing of cashflows and the likely inability to liquidate at fair value in the event of a solvency crisis, which are path-dependent events. In our example, a private equity investment can be viewed as a use of cash in early years, as the investor is required to meet capital calls, and a source of cash in later years, as distributions are realized from the investment. Additionally, existing liabilities such as spending requirements for endowments need to be considered. This necessitates a probabilistic framework to account for the unique cashflow requirements and the path dependency of each outcome in the case of portfolio insolvency.

Maintaining a desired target allocation to illiquid assets also requires investors to model the impact of liquid assets in the portfolio, as relative performance of assets can change the portfolio composition. For example, if liquid assets outperform, there may be a desire to increase illiquid assets to maintain the target allocation. However, if liquid assets underperform, selling illiquid assets can be problematic, as seen in Ang (2011) and highlighted by Siegel (2008). Selling private equity in the secondary market typically involves selling at a discount to NAV. While the discount

3 Mean-variance optimization assumes all relevant asset allocation information is captured by return, volatility, and correlation forecasts. Cashflow dynamics and the evolution of the portfolio over time are difficult to accurately capture without additional information in a mean-variance optimization. A penalty function representing disutility from allocating to illiquid assets may be used. Another recent approach discussed by Meucci (2012) is to enhance the risk estimate in a "liquidity-plus-market-risk distribution".
to NAV varies over time, in some periods (such as 2008–09), this discount can reach as high as 50%.\(^4\) (Later in this analysis, we also evaluate the impact of varying the haircut assumption on the optimal allocation to illiquid investments.)

To incorporate liquidity constraints into an asset allocation decision process, we use a simulation-based model that accounts for cashflow characteristics related to a portfolio, namely annual spending requirements (e.g., net benefit payments to retirees for pension plans) as well as outstanding commitments from private equity investments. Forward-looking returns for public bonds, public equities and private equities were stochastically generated over a period of 20 years, using the capital market assumptions detailed in Figure 1.\(^5\)

With respect to the outcomes, return is defined as the average internal rate of return (IRR) of each simulation. We use the IRR metric (rather than a time-weighted return) because of the presence of external liabilities and uneven nature of cashflows. In this framework, an efficient frontier is generated using the optimal portfolios,\(^6\) and risk is defined as the annualized volatility of IRR over the 20-year period, representing the uncertainty of outcomes. Optimal portfolios are defined as portfolios with the highest return (IRR) per unit of risk (volatility of IRR outcomes over 20 years).

Given a set of asset class assumptions, a mean-variance approach generates a single set of optimal allocations for a given level of risk tolerance. In contrast, stochastic modeling is specific to each investor, as it accounts for unique cashflows, including contributions, benefit payments and other obligations.

Private equity has structural differences in cashflow relative to public market assets (Figure 3). Contributions (use of cash) tend to occur early in the life of the investment, while the distributions (source of cash) typically start a few years after the initial investment. For the stochastic modeling approach, we assume that the portfolio has a diversified allocation to private equity funds. These cashflow schedules are averages from private equity vintages 1990–2009, which reflect cashflows from funds closed as of these respective years. (We will return to these cashflow schedules later in the paper.)

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\(^4\) Source: Cogent Partners, “Secondary Pricing Trends & Analysis,” July 2011. Private equity valuations were as of June 30, 2008; public equity valuations had also fallen significantly, so the composition of this discount may be attributed to many factors.

\(^5\) The assumptions used for calculating forward-looking asset returns are detailed in Appendix 2, page 27.

\(^6\) Realized allocations in the portfolios may not exactly match target portfolios but are generally very close to the target allocation. For example, in a simulation where the target allocation is 40% private equity, the private equity commitment schedule limits the actual amount that can be invested at any given time, such that the realized allocations may be below the target. Alternatively, the PE allocation can be modestly above target allocation due to relative outperformance.
Alternatives and Liquidity: Incorporating Liquidity Constraints into Portfolio Construction

FIGURE 3: THE CASHFLOW CHARACTERISTICS OF PRIVATE EQUITY ARE STRUCTURALLY DIFFERENT FROM THOSE OF PUBLIC EQUITY

Private equity cashflow schedule

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>


In Figure 4, we compare mean-variance optimization to a stochastic modeling approach with simplified assumptions for private equity. The charts illustrate the optimal target allocations across risk levels. We assume that the investor has no outstanding net liabilities outside of capital commitments to private equity each year, and that private equity can be sold on the secondary market at a 0% discount to NAV. In addition, we assume that the maximum allocation to private equity is 80% (i.e. before a liquidity event is triggered).

The stochastic approach results in a higher measure of risk for the same portfolio allocation. For example, the portfolio highlighted in yellow under the stochastic approach has a risk measure of 15%, and allocations of 31.6%, 59.2% and 9.2% to fixed income, public equity and private equity, respectively. Using a mean-variance approach with this same allocation, the portfolio volatility is 12.7%. Again, we point out the optimal allocations at the 10% and 15% risk targets, for illustrative purposes.

The difference in allocations is due to the increased volatility of return outcomes introduced by the timing of cashflows to and from private equity. Due to these cashflow dynamics, the volatility of outcomes in IRR is a more appropriate measure of risk, as it integrates the investor’s specific spending profile as well as the liquidity characteristics of investments. The uncertainty in the potential outcome of the portfolio with respect to solvency due to cashflow needs leads to higher risk.
The risk metric on these two charts, while analogous, is not directly comparable. On the first chart (mean-variance optimization), portfolio risk is calculated as portfolio volatility based on the underlying capital market assumptions. In the second chart (stochastic approach), the risk represents the volatility of IRR outcomes over a 20-year period.
Adding liquidity constraints

We can refine our model by incorporating an annual net payout (spending minus contributions) of 4% of initial portfolio value, maintained as a constant expenditure through time, roughly in line with an average institutional investor’s current payout rate. Figure 5 illustrates the efficient frontiers with and without liquidity considerations. The efficient frontier without liquidity considerations assumes zero net annual payout with zero secondary market transaction cost (i.e. no haircut on distressed sales). On the other hand, the efficient frontier with liquidity considerations assumes an annual net payout of 4% (of initial portfolio value) and a haircut of 50%. Both of these efficient frontiers are calculated with the stochastic modeling approach described previously. As expected, incorporating liquidity considerations into the portfolio optimization process shifts the frontier downward, particularly at high risk levels.

**FIGURE 5: ADDING LIQUIDITY CONSTRAINTS TO THE PORTFOLIO SHIFTS THE EFFICIENT FRONTIER**

![Efficient frontier graph showing the shift caused by incorporating liquidity constraints.](image)

The impact of the liquidity constraint can be directly observed by comparing the optimal portfolio allocations. Figure 6 illustrates the range of portfolio allocations across risk levels, taking into account selling at a discount and a 4% liability during liquidity events. Incorporating liquidity characteristics into the process reduces the optimal allocations to private equity compared with the mean-variance approach. This effect is more pronounced at higher risk levels. The simulation-based model effectively penalizes private equity for the additional liquidity risk.

7 A median funded corporate pension plan (represented by S&P 500 pension plans) has a net payout of approximately 5% (source: David Zion, Credit Suisse AG: Accounting Tool Box, as of 2011). Endowments have an average annual effective spending rate of about 5% (source: National Association of College and University Business Officers, 2013).
The highest allocation to private equity without accounting for the discount to NAV or annual liability was 80%; including these factors, the maximum is around 30%. At lower levels of risk, the difference is much less pronounced, as private equity allocations are relatively small. Additionally, in the simulation-based framework, the optimal allocation to fixed income also increases once liquidity considerations are incorporated as, in our example, fixed income is a proxy for a low-volatility or safe-haven asset. The reason for this is that, all else being equal, a higher allocation to fixed income reduces the volatility of outcomes in the portfolio and leads to the reduced possibility of encountering a liquidity event.

**FIGURE 6: LIQUIDITY CONSTRAINTS REDUCE THE OPTIMAL ALLOCATION OF ILLIQUID ASSETS**

Without liquidity considerations

<table>
<thead>
<tr>
<th>OPTIMAL TARGET ALLOCATIONS (%)</th>
<th>0</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXPECTED PORTFOLIO RISK (VOLATILITY OF IRR OUTCOMES) (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed Income</td>
<td>49.2%</td>
<td>50.3%</td>
<td>9.2%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public Equity</td>
<td>50.3%</td>
<td>59.2%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private Equity</td>
<td>0.5%</td>
<td>5.6%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With liquidity considerations

<table>
<thead>
<tr>
<th>OPTIMAL TARGET ALLOCATIONS (%)</th>
<th>0</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
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<tbody>
<tr>
<td>EXPECTED PORTFOLIO RISK (VOLATILITY OF IRR OUTCOMES) (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed Income</td>
<td>52.8%</td>
<td>48.6%</td>
<td>9.6%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public Equity</td>
<td>48.6%</td>
<td>50.2%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private Equity</td>
<td>0.6%</td>
<td>5.6%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: BlackRock.
Of course, there is a more practical limit as to how much investors can allocate to private equity specifically, and illiquid assets more generally. Holding all else constant, higher allocations to illiquid assets generally lead to a higher probability of a liquidity event (Figure 7).

**FIGURE 7: THE RISK OF A LIQUIDITY EVENT IS AMPLIFIED WHEN ILLIQUID ALLOCATIONS ARE HIGH**

Probability of liquidity event (20-year horizon)

Source: BlackRock. Based on a hypothetical investor with constant cash outflow equal to 4% of initial assets. Liquid assets are 60% public equities and 40% core fixed income. The time horizon is 20 years.

**SPENDING OBLIGATIONS FURTHER IMPACT ALLOCATIONS**

Investors with significant liabilities or spending requirements (relative to current assets) have a diminished capacity to allocate to illiquid assets.

Figure 8 depicts the optimal allocations to private equity at various risk levels, given a range of cash obligations. As liabilities or spending requirements increase, optimal private equity allocations tend to decrease. The higher the capital commitments, the more damaging the impact of illiquid assets may be during a liquidity crunch.
FIGURE 8: AS SPENDING REQUIREMENTS INCREASE, THE OPTIMAL ALLOCATION TO ILLIQUIDS DECREASES

Optimal allocations to private equity

Source: BlackRock.
Impact of varying input assumptions

Various inputs to the stochastic modeling process impact the optimal allocation results. Here we examine the impact of varying three parameters:

1. The expected return premium of private equity over publicly traded equity
2. The discount to NAV for selling private equity on the secondary market
3. The private equity distribution expectations (i.e. the timing of cashflows associated with the private equity investment)

1. EXPECTED RETURN PREMIUM OVER PUBLICLY TRADED EQUITY

Investors include illiquid assets in their portfolios because they expect to be compensated for the associated liquidity risk through a return premium. In this analysis, we assume a return premium of 3% over public equity (10.5%–7.5%). At higher assumed return premiums, there would tend to be higher optimal allocations to private equity, particularly for portfolios with high risk levels. The impact at low-to-moderate levels of portfolio risk (less than 15%) is much less pronounced (Figure 9).

If we assume a higher expected return to private equity investments, the optimal allocations tend to barbell between private equity and fixed income at moderate risk levels. This is due to the relatively high correlation between public and private equity, as well as the increasing risk-adjusted return features of private equity. In other words, private equity has a higher return per unit of risk, compared with public equities, when the premium is high enough. As a result, at higher risk levels, the optimal allocations tend to barbell between public and private equity. This finding can be attributed to the fact that the investor still needs a source of liquid assets to fund liabilities, and public equities have a higher standalone risk than public fixed income.

FIGURE 9: A HIGHER RETURN PREMIUM RESULTS IN BARBELL-SHAPED ALLOCATIONS

- Increasing the assumed risk premium of private equity over public equity increases the optimal allocation to private equity, particularly for portfolios with higher risk levels.
- Reducing the assumed discount sale price to NAV in secondary markets increases the optimal allocation to private equity, particularly at higher portfolio risk levels. The assumed discount to NAV has a modest impact at low-to-moderate risk levels.
- Changing the pace of asset distributions from illiquid assets has a meaningful impact on optimal allocations to private equity, especially at low-to-moderate risk levels. The more lagged the distributions, the higher the probability of a liquidity event and the lower the optimal allocation to illiquid assets.
2. DISCOUNT TO NAV

One unique characteristic of private equity investments is that a secondary market sale is often conducted at a discount to NAV, particularly in stressed scenarios. In our model, the assumed discount to NAV is an explicit penalty for illiquidity. To be conservative, we assume a 50% discount to NAV, which was observed for some assets during the financial crisis of 2008. Reducing the discount to NAV increases the optimal allocation to private equity, particularly at higher portfolio risk levels. Moving to the opposite extreme in an environment where there is no discount to NAV (i.e. secondary sales take place with no friction) we find a maximum optimal allocation to private equity of nearly 80%. That result is a function of our model, which triggers a liquidity event when private equity exceeds 80% of the total assets. Incidentally, this portfolio is also the maximum risk portfolio where liquidity events are frequent.
The assumed discount to NAV has a very modest impact at low-to-moderate risk levels, as shown in Figure 10. Because investors with modest allocations to private equity will rarely encounter a liquidity event and, therefore, will not need to sell in the secondary market, the discount for a secondary market sale has minimal impact on the asset allocation decision at these risk levels. At higher levels of portfolio risk, however, the assumed discount to NAV has a larger impact on the allocation to private equity, as an optimal portfolio seeks to balance incremental return versus the probability of a liquidity event and the ensuing discount.

**FIGURE 10: ADJUSTING THE DISCOUNT TO NAV HAS A SIGNIFICANT IMPACT AT HIGHER RISK LEVELS**

Optimal allocation to private equity

![Chart showing optimal allocation to private equity](chart)

Source: BlackRock.

3. DISTRIBUTION EXPECTATIONS (TIMING OF CASHFLOWS)

Lastly, we find that an investor’s expected horizon for drawdowns and distributions also has an impact on optimal private equity allocations. In our model, we used cashflow expectations based on the average drawdown and distributions of private equity funds with vintage years 1990–2009. To test the impact of varying the pace of distributions on the results of our asset allocation analysis, we create an alternate set of cashflow expectations to reflect an investment that takes longer to start delivering distributions (positive cashflows). To do this, we lag baseline cashflow expectations by four years and normalize the results such that the entire investment period spans 20 years.

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We find that changing the pace of distributions has a meaningful impact on results, especially at low-to-moderate risk levels. For example, at a 15% risk level, the lagged distribution results in a significantly lower optimal allocation to private equity (3.7%, compared with 9.6% for the unlagged distribution). The expected timing of cashflows from a proposed investment is a key driver of optimal allocations. The more lagged the distributions, the higher the probability of a liquidity event; therefore, as the period for distributions increases, the lower the optimal allocation to illiquid assets. Conversely, if distributions from private equity investments were accelerated, the investor’s capacity for illiquid investments would increase correspondingly. As timing of cashflows can be very difficult to accurately estimate, a more conservative schedule would lead to lower private equity allocations.

**FIGURE 11: EXTENDING CASHFLOW EXPECTATIONS HAS A SIGNIFICANT IMPACT AT LOW-TO-MODERATE RISK LEVELS**

![Graph showing the impact of lagging distributions on optimal allocations to private equity at different risk levels.](image)

Source: BlackRock. Assumes the investor has constant net liabilities equal to 4% of initial assets.
Looking at current institutional portfolios, we see a positive relationship between the allocation to illiquid assets and the size of the portfolio (Figure 12). Investors with smaller portfolios face greater challenges meeting minimum requirements for investing in multiple vintages of illiquid assets while maintaining a diversified portfolio. These investors also tend to have fewer resources to evaluate potential investments and to perform due diligence. The data indicate that, on average, investors’ allocations are broadly consistent with (though on the high end of) the ranges we identified.

Endowments and foundations typically spend 4–5% of assets each year. In 2012, the largest institutions spent 4.7%, while the smallest reported spending slightly less than 4%. These cashflows are partially offset by gifts.

By way of example, in 2012, US corporate pension funds paid out an average of 7% of assets in benefits. Pension funds also have cash inflows in the form of contributions; the average benefit payout (net of contributions) was 1.5% in 2012. Pension funds are mostly underfunded; therefore, we expect annual payout as a percent of total assets to increase over time (assuming benefits outpace asset growth). The pension funds’ lower allocations to illiquid assets likely reflect their need to fund higher cashflows in the future.9

**FIGURE 12: THE LARGER THE PORTFOLIO, THE GREATER THE CAPACITY TO INVEST IN ILLIQUID ASSETS**

Institutional investors’ allocations to illiquid assets

<table>
<thead>
<tr>
<th>SIZE OF INSTITUTION ($ AUM)</th>
<th>DB Pension</th>
<th>Endowment</th>
<th>Foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;$100m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$101-$500m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$501m-$1b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1b +</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


9 Sources: Commonfund and FactSet.
SECTION 5

A MORE RISK-AWARE APPROACH TO ILLIQUIDS

Illiquid investments, such as private equity, are an important asset class for long-term investors. But the unique characteristics of this asset class demand a more risk-aware perspective than the traditional mean-variance approach provides. Investors can address liquidity risk in the portfolio construction process with a stochastic model that incorporates their liabilities (or spending requirements), capital calls and distributions for the illiquid assets, and the potential impact of selling these assets in the secondary market.

What is an appropriate allocation to illiquid assets? The optimal level of illiquid assets depends on a number of factors. In our illustrative analysis, investors with a net annual spending obligation of 4% or less of total assets may find the optimal allocation is 15–33%,\textsuperscript{10} depending on their portfolio risk level. For investors with spending obligations higher than 10% of assets, we recommend a more modest allocation to illiquid assets, as the cost of a potential fire sale in this asset can potentially outweigh the return benefits.

What impacts the optimal level of illiquid allocations? The expected timing of distributions has the largest impact on optimal allocations at low-to-moderate risk levels (e.g. total portfolio risk of 10–15%). Prolonged distributions increase the probability of a liquidity event, so the more lagged the distributions, the lower the optimal allocation to illiquid assets. At higher risk levels, the expected return premium over public equity and the assumed discount to NAV have the most significant impact on optimal allocations to private equity; higher return premiums and lower haircuts tend to result in higher optimal allocations.

This framework may also be adapted for other illiquid assets, such as infrastructure or real estate, to incorporate unusual cashflow streams and liquidity costs into the portfolio construction process.

Stepping back from this framework, it is also important to acknowledge the significant challenge currently facing institutional investors – the need for higher yield or return. Moving into less-liquid assets may help to generate higher returns and allow for the assets to catch up to the liabilities, albeit at an increased potential for insolvency. Furthermore, constraints will be less pronounced for illiquid assets that generate regular cashflows.

\textsuperscript{10} Further discussion on optimal ranges of allocations can be found in Appendix 1, “Efficient Frontier: Acceptable Regions”.

Conclusion
APPENDIX 1

SOURCES OF CAPITAL TO MEET COMMITMENTS

The capital commitment and subsequent drawdown characteristics of illiquid assets pose a challenge to asset allocation, which typically assumes instantaneous rebalancing. By engaging in a capital commitment, the investor takes on a liability for which sources of funding must be managed. One option is to hold the committed capital completely in the cash market. The benefit to this is that the liability mismatch is minimized, as cash is immediately available. However, this results in a cash drag to the portfolio, as initial allocation levels are below the target allocation. A simple alternative approach for funding private equity commitments is to substitute directly from public equity, as the two assets share similar risk characteristics and return drivers. Special care should be taken, so that cumulative private equity commitments remain in line with target allocations; that is, private equity allocations ramp up over time and result in a steady-state allocation as earlier vintages start distributions. Note that the risk to this approach is a liability mismatch, which may occur if public equity markets experience a decline. In some instances, this means overcommitting (or undercommitting), in order to achieve a steady-state allocation in light of expected distributions as well as liabilities.

MULTI-ASSET PORTFOLIOS

In a portfolio with many asset classes that have different correlation and volatility characteristics, direct substitution of one asset for another for funding becomes very challenging. The risk profile of the portfolio must be maintained as asset allocations are ramped up, such that any biases or unintended exposures are minimized. The impact of an individual asset to the portfolio’s risk profile may be less direct and depends on the correlation of that asset with the portfolio. As a result, we propose redefining the constraint to rebalance towards desirable risk characteristics. A simple example of this would be to fund liabilities from the entire pool of liquid assets of the portfolio with the goal of keeping the total portfolio volatility consistent through time, rather than a direct substitution of one asset for another in funding. Other approaches might include matching factor exposures or incorporating additional constraints.

Target volatility is defined as the volatility of the portfolio that one seeks to rebalance towards; in this approach, we use the previous allocations and solve for the amount of capital to fund liabilities from the bond and equity assets, with the goal of holding target volatility constant throughout the horizon. We note that, in practice, pension funds that are overfunded or underfunded may have different preferences between maximizing total returns versus matching liabilities, so target volatility may also change as a function of the liability pressure itself.
PORTFOLIO VOLATILITY OVER TIME

It is also important to note that a portfolio’s volatility typically varies through time – rebalancing constraints necessitate that the portfolio volatility level changes as some assets outperform and increase relative to other allocations. (A theoretical, constantly rebalanced portfolio could achieve a constant volatility profile, but any deviation would lead to volatility fluctuating over time.) With respect to illiquid assets, this phenomenon is more relevant, as rebalancing is much more difficult to do in the short run, leading to larger potential drift in the asset allocation mix. Additionally, the possibility of fire sales in cases of insolvency will also cause jumps in the volatility profile, as more-volatile illiquid assets are exchanged for comparatively less-volatile liquid assets.

EFFICIENT FRONTIER: ACCEPTABLE REGIONS

Portfolio optimization helps to identify the set of portfolios with the highest return per unit of risk: that is, the efficient frontier. However, the optimization process is highly sensitive to input assumptions, and a portfolio just below the frontier can vary significantly from the efficient portfolio. For example, at a 15% risk level, an efficient portfolio might have a 20% private equity allocation and a total portfolio return of 5%; another portfolio immediately below the frontier might have a 30% private equity allocation and a total portfolio return of 4.7% for the same level of risk. The less efficient portfolio, while having 30 basis points lower forecast return, may be acceptable or even preferred, given the estimation errors in the input parameters and the investor’s views on the asset class.
IRR VERSUS GEOMETRIC RETURNS

In the absence of cashflows, realized annual investment returns are given by:

Annualized returns = \left( \frac{\text{Portfolio}_{\text{maturity}}}{\text{Portfolio}_{\text{initial}}} \right)^\frac{1}{\text{maturity} - 1} - 1

This relationship does not hold when there are cashflows, because the portfolio value at maturity is impacted by both cashflows and investment returns. An IRR calculation can be used to isolate the investment return, as in the simplified example below.

<table>
<thead>
<tr>
<th>Time (Yr)</th>
<th>Cash flow</th>
<th>Investment return</th>
<th>Asset value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0</td>
<td>$0</td>
<td>$100</td>
</tr>
<tr>
<td>1</td>
<td>($20)</td>
<td>$5</td>
<td>$85</td>
</tr>
<tr>
<td>2</td>
<td>($20)</td>
<td>$5</td>
<td>$70</td>
</tr>
<tr>
<td>3</td>
<td>($20)</td>
<td>$5</td>
<td>$55</td>
</tr>
<tr>
<td>4</td>
<td>($20)</td>
<td>$5</td>
<td>$40</td>
</tr>
</tbody>
</table>

Traditional return -20.47%
Internal Rate of Return (IRR) 6.36%

We calculate the risk of portfolios by evaluating the annualized standard deviation of the IRR outcomes for two reasons. First, unlike traditional portfolio risk calculated with a covariance matrix, the annualized standard deviation of the IRR outcomes accounts for the annual liability cashflows and the liquidity profile of the investor. Second, the annualized standard deviation of the IRR outcomes is a measure of dispersion in possible investment outcomes.

The annualized standard deviation of the IRR outcomes is different from the traditional portfolio risk, as demonstrated in the simplified example below that reflects a traditional portfolio with 60% equities and 40% fixed income.

<table>
<thead>
<tr>
<th>Risk measurement</th>
<th>0% Pay</th>
<th>4% Pay</th>
<th>6% Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional risk (with covariance matrix)</td>
<td>10.53%</td>
<td>10.53%</td>
<td>10.53%</td>
</tr>
<tr>
<td>Standard deviation of IRR (annual time steps)</td>
<td>11.12%</td>
<td>11.57%</td>
<td>12.92%</td>
</tr>
<tr>
<td>Standard deviation of IRR (monthly time steps)</td>
<td>10.61%</td>
<td>11.04%</td>
<td>12.13%</td>
</tr>
</tbody>
</table>

The discrepancy in the two risk measures can be attributed to two factors. First is the discretization error, which arises from the fact that simulations must be implemented in finite intervals, while geometric Brownian motion assumes assets grow at a continuous rate of return. This risk is evident when comparing the standard deviation of IRR outcomes with the traditional risk measure in the zero benefit payment scenario. The discretization error can be mitigated by using...
more granular time steps (i.e. monthly instead of annual time steps). Additionally, the magnitude of discretization error is proportional to the expected return of the portfolio and the size of the benefit payments. In general, the discretization error is a modest contributor to the incremental risk, especially in scenarios with high benefit payments.

The second and more important factor contributing to the discrepancy in risk measures is the additional risk introduced by incorporating benefit payments. Introducing benefit payments increases the dispersion in possible outcomes, as this can reduce the lifespan of simulations. Shortening the investment horizon can result in additional volatility because returns take time to converge to their theoretical or equilibrium values. For example, the long-term equilibrium annualized return of large-cap equities is expected to be 7.5%, but over a one-year time horizon, it can be anywhere from -10% to +25% based on a move of one standard deviation. This dispersion in returns is substantially reduced when the time horizon is expanded.
Monte Carlo simulations are used to evaluate the impact of incorporating liquidity into the portfolio construction process. Thus, correlated random normal returns are generated using Cholesky decomposition, and assets are assumed to follow geometric Brownian motion (GBM). All possible asset combinations are created assuming 0.50% capital increments. Each combination is simulated for 20 years; the liquid assets are rebalanced annually, assuming no transaction costs.

Benefits are assumed to be paid out annually, primarily sourced from the liquid assets. If there are insufficient liquid assets, private equity investments are sold off at a haircut to fund the benefit payments. Private equity investments are also sold when there are not enough liquid assets to fund the capital contributions.

Private equity investments are assumed to follow a static contribution/distribution schedule (see Figure 3 on page 10), with most of the contributions occurring within the first five years, and most of the distributions occurring between years four to eight. Contributions are assumed to be sourced pro rata from the liquid assets, and distributions are assumed to be invested pro rata into the liquid assets. Contributions are based on the initial private equity investment, while distributions are a function of the static schedule and the asset value over time. Private equity returns are not distinguished by funds; in other words, all private equity funds are assumed to follow the same return for a given year.

A laddered private equity investment is assumed at initiation. Additional private equity funds are added at the end of each year if the realized allocation is less than the target allocation. Conversely, private equity investments are not sold off if the realized allocation is greater than the target allocation, unless the realized allocation exceeds 80% of the total portfolio.

The internal rate of return (IRR) is calculated with the initial portfolio value, final portfolio value, and all the intermediate cashflows from the haircut cost of liquidating private equity for each simulation. The standard deviation of these IRRs is calculated and then scaled by the square root of the number of years.

A mean IRR and standard deviation of IRR is calculated for each combination. These combinations are then sorted by standard deviation of IRR. A combination is considered to be optimal if it has lower standard deviation of IRR and mean IRR than combinations with higher standard deviation of IRR.
SIMULATION DECISION PATH

Allocate assets/rebalance -> Realize returns -> Private equity distributions

- Yes

Pay liabilities and commitments

- OK

Check solvency

- Insolvent

Solve for target portfolio that maximizes return for a given level of risk, assuming annual rebalancing.

At the end of each period, capital calls are drawn from liquid assets and private equity distributions are allocated to liquid assets. Liquid assets (public equity and fixed income) are rebalanced to the correct relative allocation. If the private equity allocation is greater than 80% of total assets, a liquidity event is triggered, where a portion of the private equity is sold to bring the allocation down below 80%. We instate a threshold for fire-selling private equity when it breaches 80% because it takes some time to sell private equity on the secondary market, as investors need time to find a counterparty to take over the private equity holdings. This lag between the decision to sell and the receipt of capital necessitates that investors start selling private equity before the portfolio is fully composed of private equity, to ensure that administrative expenses and benefits for retirees are paid on time. (Lowering the threshold below 80% could result in an excessive frequency of fire sales). Outside of the haircut for private equity, no other transaction costs are assumed. Note that there may be a small difference from the model (or target allocation) compared with actual allocations, due to the rebalancing frequency to private equity. However, in our simulation, we found that the average realized allocations were nearly identical to the target allocations, for three reasons:

1. Private equity investments are “laddered in” and, thus, private equity investments at initiation are at a steady state where contributions offset distributions.

2. Individual private equity investments are assumed to have a small notional value.

3. Liquid assets are rebalanced during each period, and the transaction cost is assumed to be zero. See Appendix 2 for a detailed flow chart.
**About the authors**

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Previously, Mr. Ma covered hedge funds in the Directional Trading and Relative Value disciplines as a risk analyst in the Risk Management team for BlackRock Alternative Advisors. He began his career in 2008 in the Risk Management team for BlackRock Alternative Advisors (formerly Quellos Group, LLC) conducting quantitative research and analysis.

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Mr. Pirone's service with the firm dates back to 1997, including his years with Barclays Global Investors (BGI), which merged with BlackRock in 2009. At BGI, he was a senior client advisory strategist, advising on total portfolio investment strategy issues. Previously, he was a fixed income analyst with Gifford Fong Associates.
References


